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Damping of Hydrodynamic Modes in a Trapped Bose Gas above the Bose-Einstein Transition Temperature

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We calculate the damping of low-lying collective modes of a trapped Bose gas in the hydrodynamic regime, and show that this comes solely from the shear viscosity, since the contributions from bulk viscosity and thermal conduction vanish. The hydrodynamic expression for the damping diverges due to the failure of hydrodynamics in the outer parts of the cloud, and we take this into account by a physically motivated cutoff procedure. Our analysis of available experimental data indicates that higher densities than have yet been achieved are necessary for investigating hydrodynamic modes above the Bose-Einstein transition temperature.

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In recent experiments on magnetically-trapped atomic vapors, alkali atoms [1–3] have been cooled to temperatures at which they are degenerate and indeed Bose-Einstein condensation has been observed in them. Frequencies and damping rates of collective modes in these systems have been investigated, both above and below the Bose-Einstein transition temperature, T_c [4–6]. In this Letter we shall focus on properties above T_c. One can distinguish two regimes, the hydrodynamic one, for which the characteristic mode frequency is small compared with the collision frequency and the wavelength of the mode is large compared with the atomic mean free path, and the opposite limit, the collisionless one, for which collisions are relatively unimportant. The frequencies of modes in the hydrodynamic regime have been calculated in Ref. [7], and here we calculate their damping.

We begin by giving a simple derivation of the basic hydrodynamic equations. Our treatment is essentially that of Ref. [8] generalized to take into account the potential of the trap, and, since we are interested in small oscillations, we shall consider the linearized equations. The Euler equation for the fluid velocity $\mathbf{v}(\mathbf{r},t)$ is

$$mn_0(\mathbf{r})\frac{\partial \mathbf{v}}{\partial t} = -\boldsymbol{\nabla} p(\mathbf{r},t) + mn(\mathbf{r},t)\mathbf{f}, \tag{1}$$

where m is the mass of the atoms, $n(\mathbf{r},t)$ is the particle density, $n_0(\mathbf{r})$ is the equilibrium particle density, $p(\mathbf{r},t)$ is the pressure and $\mathbf{f}$ is the force per unit mass due to the external potential $U_0(\mathbf{r})$, $\mathbf{f} = -\boldsymbol{\nabla} U_0(\mathbf{r})/m$. In equilibrium, where the pressure is $p_0(\mathbf{r})$, Eq. (1) implies that $\boldsymbol{\nabla} p_0(\mathbf{r}) = mn_0(\mathbf{r})\mathbf{f}$. Taking the time-derivative of Eq. (1) and using the continuity equation, one finds

$$mn_0(\mathbf{r})\frac{\partial^2 \mathbf{v}}{\partial t^2} = -\boldsymbol{\nabla}\frac{\partial p(\mathbf{r},t)}{\partial t} - \boldsymbol{\nabla}\cdot[n_0(\mathbf{r})\mathbf{v}]\, m\mathbf{f}. \tag{2}$$

We calculate the first term on the right hand side of Eq. (2) by using the energy conservation condition [8],

$$\frac{\partial}{\partial t}(\rho\epsilon) = -\boldsymbol{\nabla}\cdot(w\rho\mathbf{v}) + \rho\mathbf{v}\cdot\mathbf{f}, \tag{3}$$

where ρ is the mass density and ϵ and w are, respectively, the internal energy and the enthalpy of the fluid per unit mass. Since we assume local thermodynamic equilibrium and neglect contributions to the energy due to interparticle interactions, we may use the results $p = \rho(w-\epsilon)$ and $\rho\epsilon = 3p/2$, and find that

$$\frac{\partial p(\mathbf{r},t)}{\partial t} = -\frac{5}{3}\boldsymbol{\nabla}\cdot[p_0(\mathbf{r})\mathbf{v}] + \frac{2}{3}n_0(\mathbf{r})\mathbf{v}\cdot m\mathbf{f}. \tag{4}$$

Combining Eqs. (2) and (4), and using the fact that $\boldsymbol{\nabla} n_0(\mathbf{r})$ is proportional to $\boldsymbol{\nabla} U_0(\mathbf{r})$, we obtain for the equation of motion for $\mathbf{v}(\mathbf{r},t)$,

$$\frac{\partial^2 \mathbf{v}}{\partial t^2} = \frac{5}{3}\frac{p_0(\mathbf{r})}{mn_0(\mathbf{r})}\boldsymbol{\nabla}[\boldsymbol{\nabla}\cdot\mathbf{v}] + \boldsymbol{\nabla}[\mathbf{v}\cdot\mathbf{f}] + \frac{2}{3}[\boldsymbol{\nabla}\cdot\mathbf{v}]\mathbf{f}. \tag{5}$$

Equation (5) has previously been derived by Griffin *et al.* [7] using kinetic theory.

In our present discussion we assume that the potential is axially symmetric,

$$U_0(\mathbf{r}) = \frac{1}{2}m\omega_0^2(x^2+y^2+\lambda z^2), \tag{6}$$

where ω_0 is the frequency of the trap in the $x-y$ plane; $\lambda \equiv \omega_z^2/\omega_0^2$ – where ω_z is the frequency along the z-axis – expresses the anisotropy of the trap. Most experiments on oscillations have been done in such traps, but our results may be generalized to the case of traps with no axis of symmetry. As shown in Ref. [7], in the lowest modes, which in a spherical trap correspond to monopole and quadrupole vibrations, the velocity field has the form $\mathbf{v}(\mathbf{r},t) = \mathbf{v}(\mathbf{r})\cos(\omega t)$, where $\mathbf{v}(\mathbf{r}) = a(x\hat{\mathbf{x}} + y\hat{\mathbf{y}}) + bz\hat{\mathbf{z}}$. Here a and b are constants, and the frequencies, ω, of the two modes are

$$\left(\frac{\omega}{\omega_0}\right)^2 = \frac{1}{3}\left[4\lambda + 5 \pm (16\lambda^2 - 32\lambda + 25)^{1/2}\right], \qquad (7)$$

and $b/a = 3\omega^2/2\omega_0^2 - 5$.

We turn now to the damping of these modes. We adopt the standard approach of evaluating the rate of change of the mechanical energy, E_{mech}, associated with the mode, which is given by [9]

$$\dot{E}_{\text{mech}} = -\int \frac{\kappa}{T}|\nabla T|^2 d\mathbf{r} - \int \zeta(\boldsymbol{\nabla}\cdot\mathbf{v})^2 d\mathbf{r} - \int \frac{\eta}{2}\left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i} - \frac{2}{3}\delta_{i,k}\boldsymbol{\nabla}\cdot\mathbf{v}\right)^2 d\mathbf{r}, \qquad (8)$$

where κ is the thermal conductivity, η is the first, or shear, viscosity, ζ is the second, or bulk, viscosity and T is the temperature. Because the system is inhomogeneous, the transport coefficients are generally spatially dependent. Next we show that the shear viscosity is the only source of damping of the low-lying modes described above. The contribution from thermal conduction vanishes because there are no temperature gradients for these modes, and the contribution from the bulk viscosity vanishes because ζ vanishes. To demonstrate the absence of temperature gradients, we observe that for the modes under consideration, which have the velocity field given above, $\boldsymbol{\nabla}\cdot\mathbf{v} = (2a+b)\cos(\omega t)$ is independent of position. Thus, from the continuity equation, $dn/dt + n\boldsymbol{\nabla}\cdot\mathbf{v} = 0$, it follows that $(dn/dt)/n$ is also constant. For an adiabatic process in a free monatomic gas, $n \propto T^{3/2}$, and therefore the deviation of the temperature from its equilibrium value is spatially independent. Consequently there are no temperature gradients generated by the mode, and hence no dissipation due to thermal conduction. As to the second viscosity, this vanishes because a spatially-homogeneous, non-relativistic monatomic gas in equilibrium subjected to a slow uniform dilation ($\mathbf{r} \to \nu\mathbf{r}$) remains in equilibrium, but at a different temperature. This result is independent of the statistics of the atoms and of the degree of degeneracy, and it is discussed in Ref. [10]. Inserting into Eq. (8) the expression for the velocity field, we arrive at the following simple expression for the time-average of the rate of loss of mechanical energy:

$$\langle\dot{E}_{\text{mech}}\rangle = -\frac{2}{3}(a-b)^2\int \eta(\mathbf{r})d\mathbf{r}. \qquad (9)$$

Equation (9) implies that $\langle\dot{E}_{\text{mech}}\rangle$ vanishes for the monopole mode for the isotropic case ($\lambda = 1$).

The next task is to calculate the first viscosity. At the low energies of interest in experiments, the scattering of two atoms is purely s-wave, and the total cross section is $\sigma = 8\pi a_{\text{scat}}^2$, where a_{scat} is the scattering length. We consider the viscosity in the classical limit, since we expect the classical limit to be quantitatively accurate, even at T very close to T_c, from comparison to the known effects of degeneracy on the heat capacity. The viscosity has the general form [10]

$$\eta = C_\eta\frac{(mkT)^{1/2}}{\sigma}. \qquad (10)$$

A simple relaxation-time approximation, with a scattering rate equal to $n_0(\mathbf{r})\sigma v$, where $v = (2\epsilon/m)^{1/2}$ is the particle velocity and ϵ is the single-particle energy, leads to the result $C_\eta = 2^{7/2}/(15\pi^{1/2}) \approx 0.426$, while a variational calculation gives $C_\eta = 5\pi^{1/2}/2^4 \approx 0.554$.

An important feature of the expression for the viscosity is its independence of the particle density. Consequently, the integral in Eq. (9) formally diverges at large distances from the center of the trap. The reason for this is that hydrodynamics fails, because in the outer parts of the cloud, particle mean free paths are too long for hydrodynamics to be applicable. To solve this problem, one should treat the outer parts of the cloud using kinetic theory, rather than hydrodynamics, but to obtain a first estimate of the effects we shall assume that the hydrodynamic description holds out to a distance such that an atom incident from outside the cloud has a probability of no more than $1/e$ of not suffering a collision with another atom. In the parlance of radiative transport, this corresponds to an optical depth of unity. Mathematically, this condition is

$$1 \approx \int_{r_{0,s}}^{\infty}\frac{ds}{l(\mathbf{r})}, \qquad (11)$$

where the local mean free path is $l(\mathbf{r}) = [n_0(\mathbf{r})\sigma]^{-1}$, and $r_{0,s}$ is the cutoff, which depends on direction. The integral in Eq. (11) is to be performed along the path for which the density gradient is steepest, that is along $\boldsymbol{\nabla}U_0$, and ds is the corresponding line element. For a classical distribution, $n_0(\mathbf{r}) = n(0)e^{-U_0(\mathbf{r})/k_BT}$, where $n(0)$ is the density at the center. Since $dU_0 = |\boldsymbol{\nabla}U_0|ds$, Eq. (11) can be written as

$$1 \approx n(0)\sigma k_BT\frac{1}{|\boldsymbol{\nabla}U_0(\mathbf{r}_{0,s})|}e^{-U_0(\mathbf{r}_{0,s})/k_BT}. \qquad (12)$$

For large values of the dimensionless parameter $\lambda^{1/2}Na_{\text{scat}}^2 m\omega_0^2/k_BT$, Eq. (12) gives

$$r_{0,s}^2 \approx 2\frac{k_BT}{m\omega_0^2}\frac{1}{\sin^2\theta + \lambda\cos^2\theta}\ln\tau_s(\theta), \qquad (13)$$

where θ is the angle of $\mathbf{r}_{0,s}$ with respect to the z-axis. The dimensionless quantity $\tau_s(\theta)$ is essentially the total optical depth at the center of the cloud

$$\tau_s(\theta) \equiv \tau_0\lambda^{1/2}\left(\frac{\sin^2\theta + \lambda\cos^2\theta}{\sin^2\theta + \lambda^2\cos^2\theta}\right)^{1/2}, \qquad (14)$$

where

$$\tau_0 = \sigma n(0)\left(\frac{k_BT}{2m\lambda\omega_0^2}\right)^{1/2}. \qquad (15)$$

The density at the center of the cloud is $n(0) = N\lambda^{1/2}(m\omega_0^2/2\pi k_BT)^{3/2}$. The volume of the atomic cloud

is given by integrating $r_{0,s}^3/3$, Eq. (13), over the solid angle. Equation (9) with the variational estimate for the viscosity ($C_\eta = 5\pi^{1/2}/2^4$) can then be written as

$$\langle \dot{E}_{\rm mech} \rangle \approx -\frac{5\pi^{1/2}}{2^{7/2}3^2}(a-b)^2 \frac{(k_B T)^2}{m\omega_0^3 a_{\rm scat}^2} f(\lambda, \tau_0). \quad (16)$$

The function $f(\lambda, \tau_0)$ is defined as

$$f(\lambda,\tau_0) = \int_{-1}^{1} \frac{dx}{[(\lambda-1)x^2+1]^{3/2}} \times \times \left[\ln\left(\tau_0 \lambda^{1/2} \left(\frac{(\lambda-1)x^2+1}{(\lambda^2-1)x^2+1} \right)^{1/2} \right) \right]^{3/2}. \quad (17)$$

We expect the leading term ($\propto [\ln \tau_0]^{3/2}$) to be asymptotically exact for large τ_0, but the value of the cutoff in the logarithm will depend on the detailed kinetic-theory solution to the boundary-layer problem.

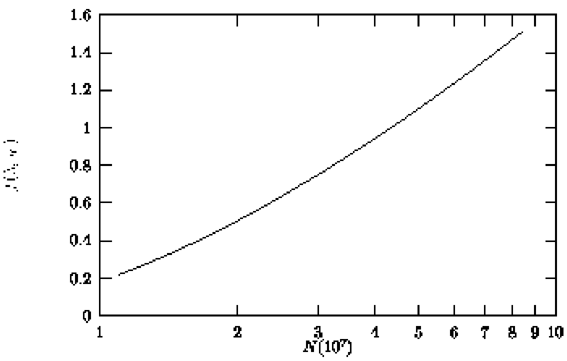

FIG. 1. We plot $f(\lambda, \tau_0)$ from Eq. (17) as function of the number of particles N, for $\lambda = 8$, $a_{\rm scat} = 53$ Å, $\nu_0 = 129$ Hz, and $T = 1.3\,T_c$, which are the parameters in the experiment of Ref. [6]. Asymptotically, for $\tau_0 \gg 1$, $f(\lambda = 8, \tau_0) \approx 2\lambda^{-1/2}[\ln(\tau_0\lambda^{1/2})]^{3/2} - 0.53\,[\ln(\tau_0\lambda^{1/2})]^{1/2}$.

To calculate the damping rate we need to evaluate the mechanical energy of the cloud, $E_{\rm mech}$. In any oscillator the time-average of $E_{\rm mech}$ is equal to the maximum kinetic energy in the mode, so

$$\langle E_{\rm mech} \rangle = \frac{1}{2} \int m n_0(\mathbf{r}) v^2(\mathbf{r}) d\mathbf{r}, \quad (18)$$

and therefore,

$$\langle E_{\rm mech} \rangle = N \frac{k_B T}{\omega_0^2} \left(a^2 + \frac{b^2}{2\lambda} \right). \quad (19)$$

We can now introduce the *amplitude* damping rate $\tau_{\rm damp}^{-1}$ (as opposed to the *energy* damping rate, which is twice the amplitude damping rate), given by the absolute value of the ratio $\langle \dot{E}_{\rm mech} \rangle / 2\langle E_{\rm mech} \rangle$,

$$\tau_{\rm damp}^{-1} \approx \frac{5\pi^{1/2}}{2^{9/2}3^2} \frac{k_B T}{m\omega_0 a_{\rm scat}^2} \frac{1}{N} \frac{(a-b)^2}{(a^2 + b^2/2\lambda)} f(\lambda, \tau_0). \quad (20)$$

In order to elucidate the origin of the damping of the modes given by Eq. (7), we remark that when the viscosity is taken to be constant, the non-equilibrium stress tensor is constant in space, and therefore does not give rise to any force on a fluid element. The damping is caused by the entropy generated by the non-equilibrium energy current density, which has a constant, non-zero divergence.

Let us now turn to the experiments on sound propagation that have been performed to date. A necessary condition for hydrodynamics to be applicable is that the mean free path be small compared with the characteristic length scale of the mode. For the low modes we are studying here, this is equivalent to the requirement that the optical depth $\tau_s(\theta)$ be large compared with unity in all directions. For the MIT experiment [4], for which $\lambda < 1$, this condition implies that $\lambda^{1/2}\tau_0 \gg 1$. In terms of the characteristic lengths, $R_z = (2k_B T/m\omega_z^2)^{1/2}$ and $R_\perp = (2k_B T/m\omega_0^2)^{1/2}$, which measure the spatial extent of the cloud along the z-axis and perpendicular to it, the condition $\lambda^{1/2}\tau_0 \gg 1$ is seen to be equivalent to the requirement $R_\perp \gg l(0)$, where $l(0)$ is the mean free path in the center of the cloud. For the parameters of this experiment [4], the minimum number of particles that is required for this condition to be satisfied is $\approx 3 \times 10^9$, whereas the number of particles at $T = 2\,T_c$ is found to be $N \approx 5 \times 10^7$ [11]. For the parameters in the JILA experiment [5,6], the minimum value of $\tau_s(\theta)$ is τ_0, since $\lambda > 1$. In this case the condition $\tau_0 \gg 1$ is equivalent to $R_z \gg l(0)$. Therefore, for the JILA experiment, the minimum number of particles required to attain hydrodynamic conditions is $\approx 1 \times 10^7$ at $T = 1.3\,T_c$, which is to be compared with the experimental value $N \approx 8 \times 10^4$ at $T \approx 1.3\,T_c$ [6]. Figure 1 shows $f(\lambda, \tau_0)$ as function of N, with all the other parameters equal to the ones of Ref. [6] and $T = 1.3\,T_c$. The lowest value of N is chosen to be $N = 1.1 \times 10^7$, i.e., the one for which $\tau_0 = 1$.

Now we estimate for the MIT and JILA experiments the magnitude of the characteristic lengths R_z and $R_\perp$, and compare them with the mean free path in the center of the cloud. The transition temperature is obtained from $k_B T_c = 0.94\, N^{1/3}\hbar(\omega_0^2\omega_z)^{1/3}$. For the MIT experiment $N \approx 2.5 \times 10^7$ at T_c [11], while $\omega_0 \approx 2\pi \times 250$ Hz and $\lambda \approx 5.8 \times 10^{-3}$ [4]. At $T = 2\,T_c$ the particle number is $\approx 5 \times 10^7$, resulting in $R_z \approx 380$ μm and $R_\perp \approx 29$ μm. The corresponding value of the central density $n(0) = N/(\pi^{3/2} R_z R_\perp^2)$ is $\approx 2.9 \times 10^{13}$ cm^{-3}. With the scattering cross section $\sigma \approx 1.9 \times 10^{-12}$ cm^2 ($a_{\rm scat} \approx 28$ Å) [12], we obtain $l(0) = [n(0)\sigma]^{-1} \approx 180$ μm, which is less than R_z, but much larger than $R_\perp$. The other condition for hydrodynamic behaviour is that the frequency of the mode be small compared with an average particle scattering rate. A single particle undergoes collisions at a rate $n_0(\mathbf{r})\sigma v_{\rm th}$, where $v_{\rm th} = (8k_B T/\pi m)^{1/2}$ is the average particle velocity. The average scattering rate in the cloud is thus

$\tau_{\rm scat}^{-1} = \int n_0^2(\mathbf{r})\sigma v_{\rm th}\, d\mathbf{r}/\int n_0(\mathbf{r})\, d\mathbf{r} = n(0)\sigma(k_BT/\pi m)^{1/2}$, since $\int n_0^2(\mathbf{r})\, d\mathbf{r}/\int n_0(\mathbf{r})\, d\mathbf{r} = n(0)/2^{3/2}$. For the MIT experiment we therefore estimate $\omega\tau_{\rm scat} \approx 2.2$ at $T = 2\,T_c$.

For the JILA experiment $\omega_0 \approx 2\pi \times 129$ Hz, $\lambda = 8$ and the particle number at T_c is $N \approx 4 \times 10^4$ [6]. At $T = 1.3\,T_c$, where $N \approx 8 \times 10^4$, we obtain $R_z \approx 4$ μm and $R_\perp \approx 10$ μm, while the central density is $n(0) \approx 3.7 \times 10^{13}$ cm^{-3}. The mean free path $l(0)$ is estimated to be 38 μm corresponding to a scattering cross section $\sigma \approx 7 \times 10^{-12}$ cm^2. Finally the dimensionless parameter $\omega\tau_{\rm scat}$ is ≈ 19. We should mention that using Eq. (20) we find that for $N = 1.1 \times 10^7$, the amplitude damping times for the JILA experiment are ≈ 8 ms and 47 ms at $T = 1.3\,T_c$ for the modes which correspond to monopole and quadrupole vibrations in a spherical trap, respectively.

We thus conclude that even though in the MIT experiment $l(0) \lesssim R_z$, there are as yet no experiments with which we can directly compare our calculation of the damping. To obtain a semi-quantitative description, we adopt a phenomenological interpolation formula for the frequency and the damping rate of the modes. This has the usual form,

$$\omega^2 = \omega_C^2 + \frac{\omega_H^2 - \omega_C^2}{1 - i\omega\tau}, \tag{21}$$

characteristic of relaxation processes. Here ω_C is the frequency of the mode in the collisionless regime and ω_H in the hydrodynamic regime, and τ is a characteristic relaxation time, which we anticipate will be of order the scattering time. Equation (21) gives the qualitatively correct limiting behaviour for $\omega\tau \gg 1$ and $\omega\tau \ll 1$. The imaginary part of the above equation gives for the damping rate

$$\tau_{\rm damp}^{-1} = \frac{1}{2}\frac{(\omega_C^2 - \omega_H^2)\tau}{1 + (\omega\tau)^2}. \tag{22}$$

The amplitude damping time in the MIT experiment [4] is measured to be ≈ 80 ms at $T = 2\,T_c$. The frequency $\nu_C = 2\nu_z \approx 38$ Hz; Eq. (7) gives for $\nu_H \approx (12/5)^{1/2}\nu_z \approx 30$ Hz. Using these numbers and Eq. (22), we can solve for $\omega\tau$ to get the two solutions 3.6 and $3.6^{-1} \approx 0.28$. We regard the larger solution as being the physically relevant one, since it is close to our estimate of $\omega\tau_{\rm scat} \approx 2.2$. The real part of Eq. (21), with $\omega\tau = 3.6$, implies that the oscillation frequency is ≈ 37.5 Hz, which is consistent with the experimental value of 35 ± 4 Hz. For the JILA experiment [6], the amplitude damping time is ≈ 50 ms at $T \approx 1.3\,T_c$. Since $\nu_C \approx 258$ Hz and $\nu_H \approx 221$ Hz, Eq. (22) implies that $\omega\tau \approx 10.6$ or 10.6^{-1}. Again we consider the larger value to be the physical one, since it is close to our theoretical estimate above, $\omega\tau_{\rm scat} \approx 19$. We thus conclude that damping of the modes in clouds of bosons above T_c is in good agreement with theoretical expectations.

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